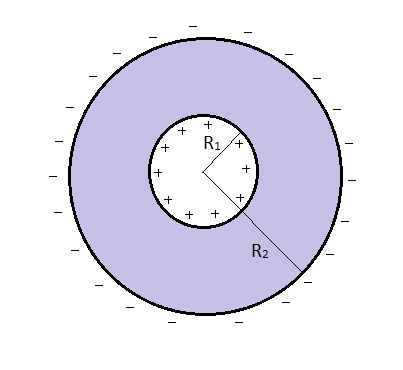
Resistor Problems

**Question 1.** A spherical capacitor (R1 = 3mm, R2 = 5mm) is discharging radially outward through its dielectric. The dielectric has resistivity of 48μΩ∙m. If the current passing radius r = 4mm is 7.8A, what is the electric field strength at this radius?



We have from Ohm’s law: I = ΔV/R = ΔV/[ρL/A] = (ΔV/L)∙A/ρ = EA/ρ. So E = Iρ/A = (7.8)(48\*10-6)/(4π∙0.0042) = 1.9 V/m.

**Question 2.** With respect to the above problem, if the speed of charges at the outer radius is v = 17μm/s, what is their speed at the inner radius?

The currents at either radius must be equal, and so we have:

I1 = I2

nev1A1 = nev2A2

v1 = v2A2/A1 = v2(4πr22/4πr12) = v2(r2/r1)2 = (17μm/s)(5mm/3mm)2 = 47.2μm/s

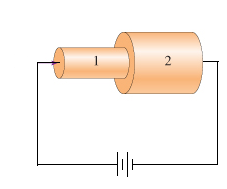
**Question 1**. The electron beam inside a television picture tube is 0.50 mm in diameter and carries a current of 65 μA. This electron beam impinges on the inside of the picture tube screen. If the electrons are traveling at a speed of v = 3×107 m/s, what power is delivered to the screen?



**Question 4**. Over time, atoms "boil off" the hot filament in an incandescent bulb and the filament becomes thinner. How does this affect the brightness of the lightbulb and why (be as explicitly clear as possible).

As the wire gets thinner, its resistance gets larger. The potential difference across the wire doesn’t change. The power dissipated is P = IΔV = (ΔV/R)(ΔV) = (ΔV)2/R. Since R goes up, P goes down, and the brightness decreases.

**Question 4**. Two resistors are hooked up to a battery. They are made of identical material and are of identical length. Resistor 2 is obviously wider than resistor 1. So….(a) is I1 greater than, less than or equal to I2? (b) is the electric field in resistor 1 greater than, less than, or equal to the electric field in wire 2? (c) is the electron drift velocity in wire 1 greater than, less than, or equal to the drift velocity in wire 2?



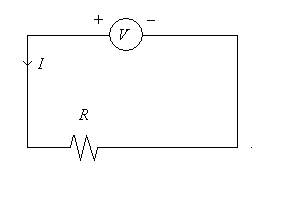
(a) I1 = I2 by charge conservation.

(b) E = ΔV/ℓ = IR/ℓ= I(ρℓ/A)/ℓ = Iρ/A. Since A is larger for wire 2, E is smaller for wire 2.

(c) I = nevA → v = I/neA. Again, since A is larger for wire 2, v is smaller for wire 2.

**Problem**

Consider the following circuit with V = 10V, and R = 20Ω.



What is the current in the circuit?

**Solution**

Well from Ohm’s law,



The power supplied by the battery is:



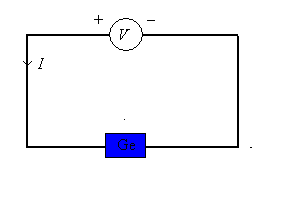
The power dissipated by the resistor is:



and so we see that the power supplied by the battery is equal to the power dissipated by the resistor – as energy conservation would demand.

**Problem**

Suppose we have the following circuit. We connect a battery (V = 10V) to a germanium resistor (ρGe = 0.46Ωm, density = 5500kg/m3, atomic mass = 72.6g/mol), via two copper wires (ρCu = 1.68×10-8 Ωm, density = 9000kg/m3, atomic mass = 63.5g/mol). Suppose the two wires each have a length of L of 10cm, and a cross section of A = 3.14×10-6m2, which is typical. And suppose the Ge resistor has a length of 1cm, and cross sectional area A = 3.14×10-5m2.



What is the current in the circuit? What is power supplied by battery? What is power dissipated in one of the Cu wires? In one of the Ge resistor? What is average electron velocity? What is collision time τ? What is E field in the Cu wires? What is E field in the Ge resistor?

**Solution**

Well, the total resistance of the circuit is just the sum of the resistances of the individual pieces – the two wires and the Ge resistor. The resistance of a single wire is:

mΩ



and the resistance of the Ge resistor is:



So the total resistance of the circuit will be:



As we see, the resistance of the metal wires is so much smaller than the resistance of the ‘resistor’ that we usually just ignore the resistance of the wires. The current is:



What is the power supplied by the battery?

.

What is the power dissipated by one of the Cu wires? .

What is the power dissipated by the Ge resistor?



So note that the power supplied by the battery is equal to the power lost in the circuit. Conservation of energy would require this of course.

Now let’s consider what the typical velocity of e+’s in the circuit is. One might think that this velocity would be extremely fast, but in fact it is quite slow. For example, let’s consider the current running through the Cu wire above. From above, the velocity of the e+’s is:



where nc is the charge density of the copper wire, and A is the cross section area of the copper wire. We can calculate nc for copper in the following way. nc is the amount of charge per unit volume in the copper wire. Suppose that each copper atom in the wire contributes one e+ to the curren, then nc =e+(# Cu atoms per unit volume), where e+ is 1.6×10-19C. Then we have,



where NA is Avogadro’s number = 6.02×1023, Cumolar mass is the molar mass of Cu, i.e., 63.5g, and ρCu is the density of copper = 8.94g/cm3. Therefore, nc is:



filling this into the formula for vave, we have,



So the electrons in a typical circuit traverse the circuit at a *very* slow pace. We would like to solve for τ in the copper wires above. So using the formula for the resistivity:



Filling in the values for Cu, we have,



so the e+’s collide with an impurity every 25ns! This is why the e+ do not go very fast in the wire – they don’t accelerate long before they’re knocked back to rest.

What is the strength of the electric field in the Cu wires? Consider the left hand wire; its length is 10cm, and the potential difference across it is:



From ΔV = -**E**·Δ**r**, the field *strength* would be:



so we see that the E field in the copper wires is extremely weak. What about the resistor?

What is the strength of the electric field in the Ge resistor? The potential difference across it is:



From ΔV = -**E**·Δ**r**, the field *strength* would be:



so we see that the E field in resistor is much larger by comparison. This must be the case, because if we are to have the same current in the wires as in the resistor – as we must – then the E field must be much larger in the resistor than in the wires since it takes much more force to move the e+’s through the Ge resistor than it does to move them through the Cu wires.

**Example**

Suppose battery (6V) is hooked up across a copper wire with d = 3mm, L = 20cm, ρ = 2×10-8 Ωm, density = 9000kg/m3, atomic mass = 63g) What is R, ΔV, I, P, E, v, τ?

R = ρL/A = (2×10-8)(0.20)/(π∙0.00152) = 0.566mΩ.

ΔV = 6V

I = ΔV/R = 10600 A

P = IR2 = 63400 W

E = ΔV/L = 6/0.20 = 30V/m

v = I/neA = 10600/[2.96∙1028∙1.6∙10-19∙π(0.00152)] = 0.317m/s

τ = v/a = v/(F/m) = mv/F = mv/eE = 9.11×10-31∙0.317/1.6×10-19∙30 = 6×10-14­ s.

**Example**

Suppose have current I = 3A running through wire (d = 3mm, L = 20cm, ρ = 2×10-8 Ωm, density = 9000 kg/m3, atomic mass = 63g), tungsten filament (d = 0.03mm, L = 10cm, ρ = 5×10-8 Ωm, density = 1900 kg/m3, atomic mass = 183g), and wire (d = 3mm, L = 20cm, ρ = 2×10-8 Ωm) again. What is the current in tungsten filament? What is current in tungsten? What are velocities? What are field strengths? What are potential differences?

R = ρL/A = (2×10-8)(0.20)/(π∙0.00152) = 0.566mΩ.

I = 0.75A

ΔV = IR =

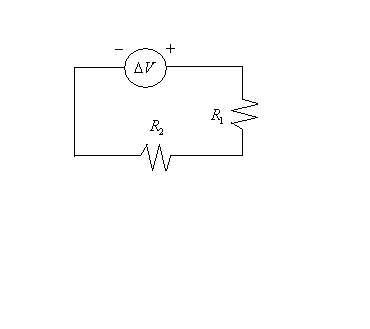
E = ΔV/L = 6/0.20 = 30V/m

v = I/neA = 10600/[2.96∙1028∙1.6∙10-19∙π(0.00152)] = 0.317m/s

τ = v/a = v/(F/m) = mv/F = mv/eE = 9.11×10-31∙0.317/1.6×10-19∙30 = 6×10-14­ s.

**Example**

What is the currrent through the battery? Let ΔV = 10V, and R1 = 1Ω and R2 = 2Ω.



**Solution**

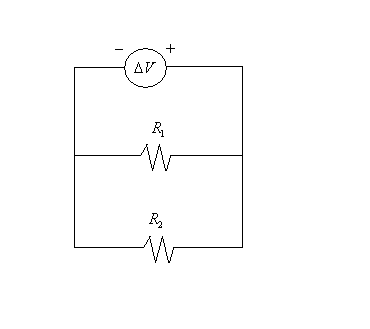
The resistors are in series so Req. = R1 + R2 = 3Ω. So the current through the battery is:



This is also the current through each individual resistor.

**Problem**

What is the current through the battery? and through each individual resitor? Let ΔV = 10V, and R1 = 1Ω and R2 = 2Ω.



**Solution**

The equivalent resistance is:



Therefore the current through the battery is:



The current through R1 is:

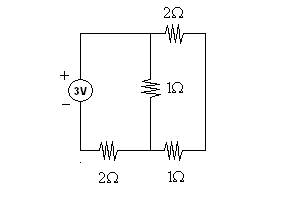


and the current through R2 is:

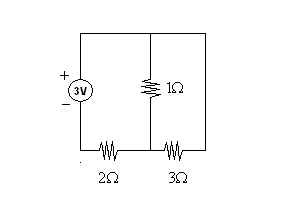


Note that ΔV1 and ΔV2 = 10V, the potential difference across the battery, because they are connected directly across the battery. Also note that the current going through the battery splits up into I1 and I2, so naturally I1 + I2 = 15A, the current through the battery.

1. What is the equivalent resistance of the circuit below? What is the power dissipated across the 2Ω resistor in the lower left hand corner?

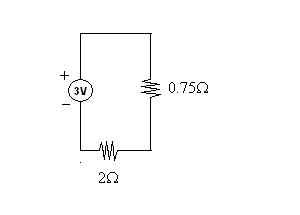


Can combine the two on right in series,

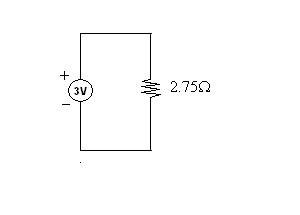


and then the right two in parallel:





and now combine in series,



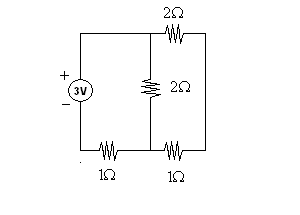
So 

We need the current running through the 2Ω resistor. This is just the current running through the battery, which is: I = ΔV/R = 3/2.75 = 1.09A. Therefore the power dissipated in the 2Ω resistor is:



**Problem**

What is the equivalent resistance of the circuit below? What is the power dissipated across the upper 2Ω resistor?

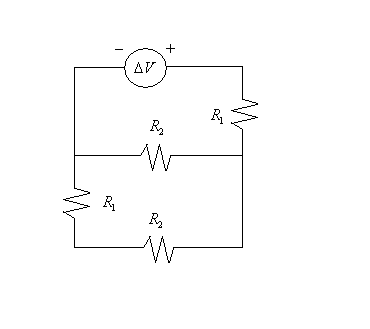


**Solution**

The 2Ω and 1Ω are in series to give 3Ω. Then this can be combined in parallel with the 2Ω resistor to give R = [1/3 + 1/2]-1 = 6/5 = 1.2Ω. And then this can be combined in series with the 1Ω to give Req. = 2.2Ω. Working backwads, across the 2.2Ω resistor is I = 3/2.2 = 1.36A. This resistor expands in series into the 1Ω and 1.2Ω resistors. Therefore the current is the same across these. So the current across the 1.2Ω is also 1.36A. Then the 1.2Ω expands in parallel into the 2Ω and 3Ω resistors. Thus the potential difference remains the same. The potential difference across these is ΔV = (1.36)(1.2) = 1.63V. Now the 3Ω resistor expands in series into the 1Ω and 2Ω resistors. Thus the current remains the same. The current is I = ΔV/R = 1.63V/3Ω = 0.54A. And the power dissipated is: P = I2R = (0.54)2(2) = 0.58W.

**Problem**

Consider the following example,

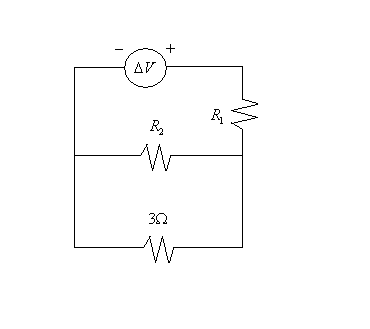


Calculate the equivalent resistance, current flowing through the battery, current flowing through each of the branches. Total power dissipated. Let ΔV = 10V, and let R1 = 1Ω, R2 = 2Ω.

**Solution**

Then we’ll start by combining bottom R1 and R2 in series,

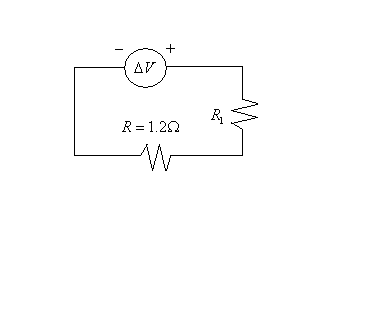




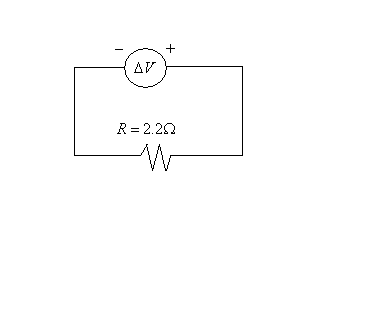
Now R2 and 3 are in parallel, so,



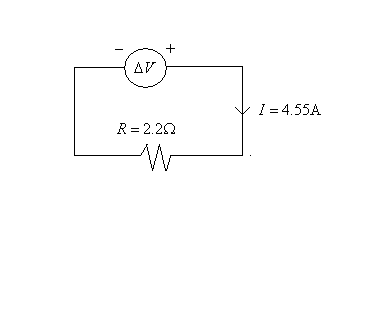
So we have,



and now these are in series. Req = 1.2 + 1 = 2.2Ω, so,



So the equivalent resistance of the circuit is 2.2Ω. Therefore the current through the battery is: I = ΔV/R = 10/2.2 = 4.55A.



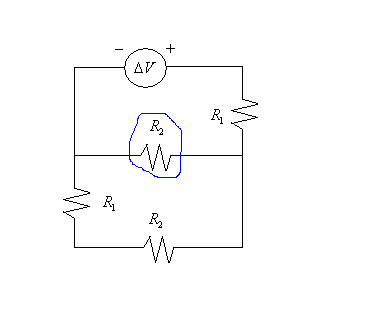
And the total power supplied by the battery is therefore,



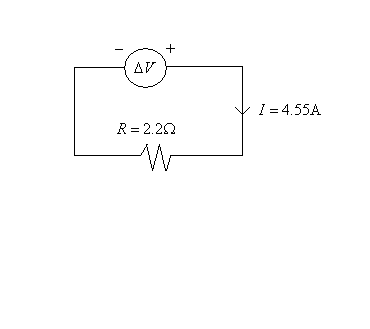
The total power dissipated by the circuit is:



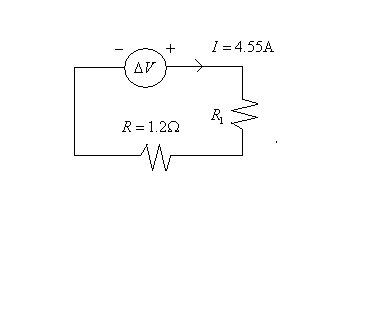
Of course they must be the same because of energy conservation. Now, let us ask what the current is in the circled resistor.



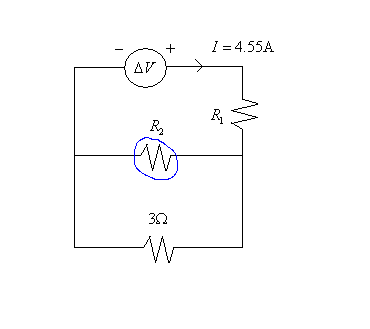
Well, we work backwards like with capacitors. Starting from the equivalent circuit,



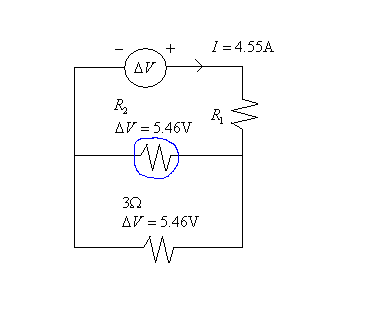
We expand the 2.2 in series to get the 1.2 and the 1. Since we expanded in series, the 2.2 equivalent, and the 1.2, 1 constituents all have the same current. Therefore, we have,



Now, we expanded the 1.2 in parallel to get R2 and the 3,



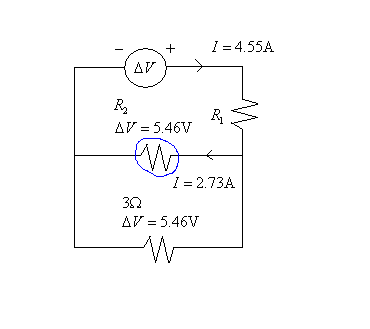
Since we expanded in parallel, it is the potential difference which will remain the same between the 1.2, and its R2 and 3 constituents. We must therefore calculate the potential difference across the 1.2. By Ohm’s law is is ΔV = IR = (4.55)(1.2) = 5.46. Therefore we have:



Now it was R2 for which we were trying to find the current. Since we know ΔV and R, we use Ohm’s law to conclude,



So we have the current through R2 is 2.73A, illustrated below,

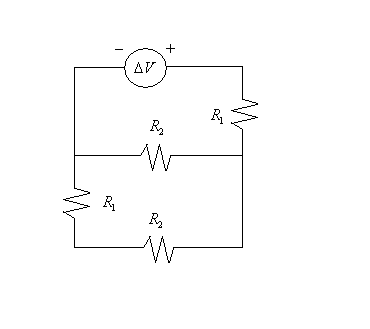


which is what we wanted to know. We could calculate the power dissipated across R2. This would be,



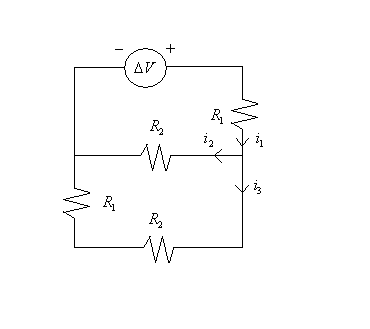
**Problem**

Start with example can do using equivalent resistance. For comparisons sake, we’ll take ΔV = 10, and R1 = 1, R2 = 2. What are currents in wires?



**Solution**

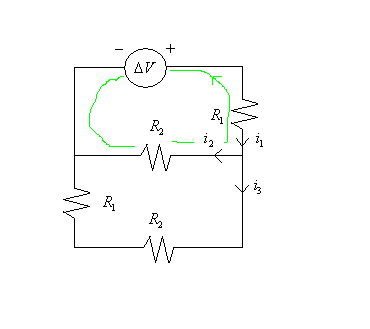
First we label currents in arbitrary directions,



KCL requires:



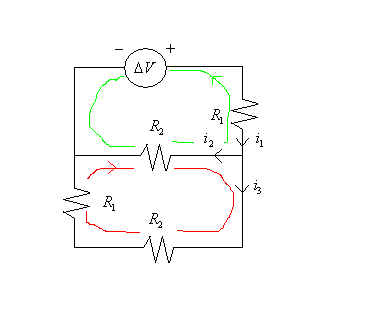
Next we make a closed loop around the circuit and apply KVL. Consider the following loop,



Then starting at the battery we have,



and now the final equation. We’ll choose another loop – the bottom half.



KVL applied to this loop is:



Plugging the KCL into Eq.2 gives us,



Plugging it into Eq.3 does nothing,



To solve these two simultaneous equations, we will solve for i3 in Eq. 2 and plug that into Eq. 3. So,



and so,



Note that this is precisely what we obtained before in the previous lecture. Continuing, we can now solve for i3,

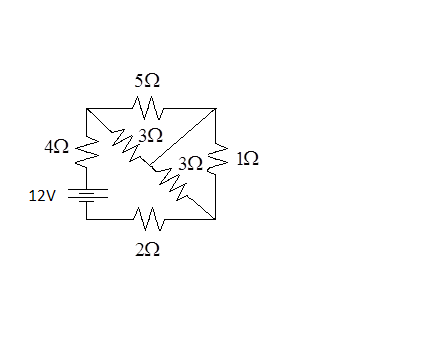


and the last current,



which is also what we obtained before.

**Question 5**. What is the power dissipated by the 5Ω lightbulb?



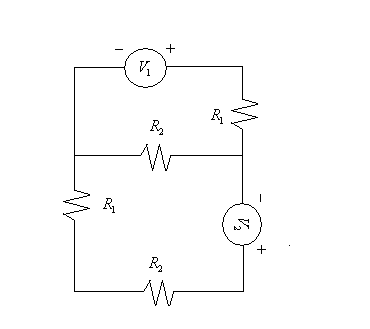
Equivalent resistance is:



and so the current through the battery is I = 12/8.63 = 1.4A. Then consider KCL at the top junction: we have 1.4 = i5 + i3. And consider KVL around the top loop: -5i5 + 3i3 = 0 → i3 = 1.67i5. Plugging this into the KCL we have: 1.4 = I5 + 1.67i5 → 1.4 = 2.67i5 → i5 = 0.52A. And so P5 = i52R5 = (0.52)2(5) = 1.35W.

**Problem**

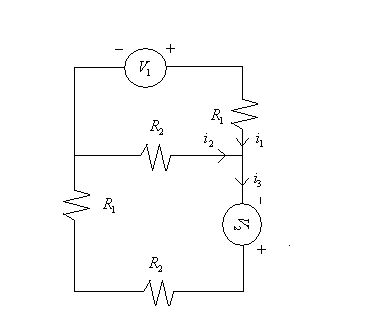
Now apply to something like,



In this case, we cannot reduce the circuit to an equivalent resistance, and so we’ll have to use Kirchoff’s laws to determine the currents. Preliminarily, we’ll let V1 = 10V, V2 = 20V, and R1 = 1Ω, R2 = 2Ω as before. Want currents in all wires.

**Solution**

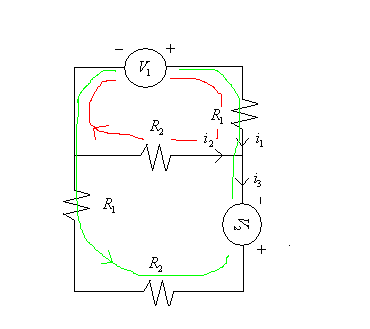
Then to start, we again will write in arbitrary currents,



KCL gives our first equation,



Next we will apply KVL to the top loop,



Then we’ll write out KVL for the red loop,



And then for the green loop.



Next we plug Eq.1 into Eq.2 and Eq.3,



Eq.2 is unaffected, but for Eq.3 we have



Then solve for i1 with Eq.2 perhaps,



and plug into Eq.3



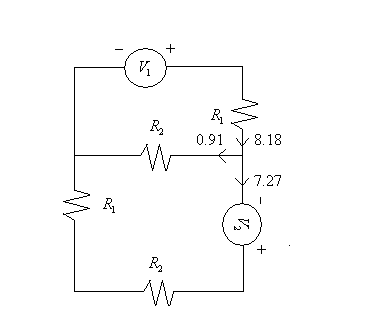
So i2 = -0.91. And now we plug this into Eq.2 to get i1,



and now plug these into Eq.1 to get i1,



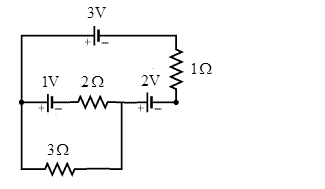
Notice that we obtained a negative result for i2. That simply means that the current is going in the opposite direction to that guessed. So we have actually,



\* Note that it is completely irrelevant which way we guess the currents, and which loops we traverse, as well as which direction we traverse the loops. We will still get the same results in the end.

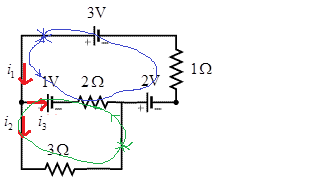
**Problem**

For the circuit below, calculate the magnitude *and* direction of the current running through the 2V battery.



**Solution**

Label currents in the circuit i1, i2, i3.



Then KCL says that i1 = i2 + i3.

KVL around the blue loop says that -1 – 2i3 – 2 – 1i1 + 3 = 0 → i1 = -2i3.

KVL around the green loop says that +2i3 + 1 – 3i2 = 0 → 3i2 - 2i3 = 1.

Plugging the KCL equation into the bottom two yields…



Solving for i2 in the top equation and plugging into the bottom yields.



and therefore we have:



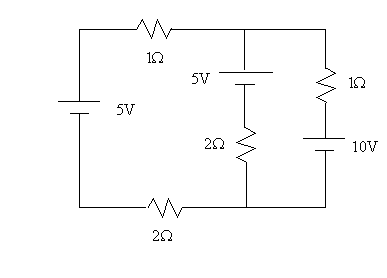
and finally,



So the current running through the 2V battery is 2/11 Amps, to the right.

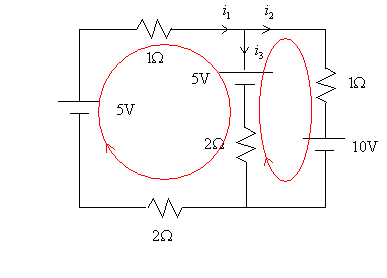
**Problem**

Calculate the power dissipated in the top 1Ω resistor.



**Solution**

Drawing currents and loops as shown:



KCL gives:



KVL on the left gives:



and KVL on the right gives:



Plugging the KCL into first KVL gives:



Plugging this into second KVL gives:



and therefore for i3 we have:



and therefore for i1 we have:

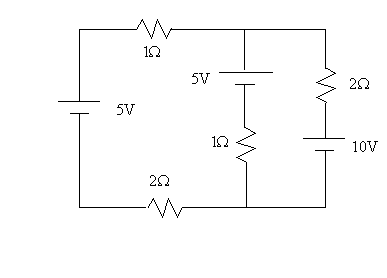


And so the power dissipated through the upper left 1Ω resistor is:



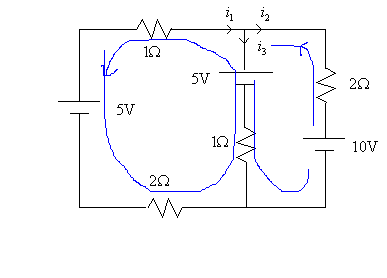
**Problem**

Calculate the magnitude and direction (up or down) of the current flowing through the 10V battery.



**Solution**

Label currents, and draw two paths.



current conservation (KCL) at the top junction requires,



(KVL) around the left loop gives,



(KVL) around the right loop gives,



Plugging 1 into 2 and 3 gives,





Plugging 2 into 3 gives,



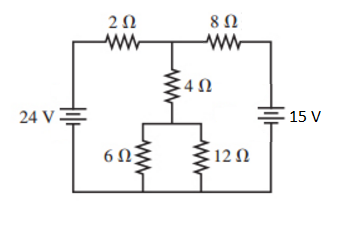
which means



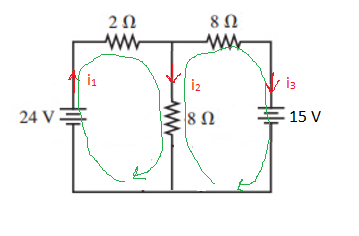
therefore the current is



**Question 3.** What is the power dissipated by the 8Ω lightbulb?



First we should combine the resistors in the middle wire into one equivalent. Req = (6-1 + 12-1)-1 + 4 = 8.



So then we have the circuit above. Now label currents i1, i2, i3, and write out KCL at the junction, and KVL for the two loops.



Filling the top into the middle and bottom equations we have:



Now solve for i3 in first equation and substitute into the last equation:



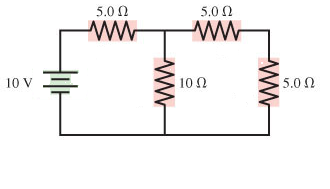
And so i3 is:



and so the power dissipated by the 8Ω lightbulb is:

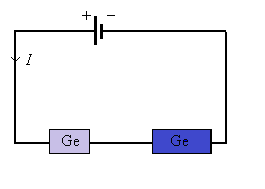


**Question 4**. What is the current in the middle 10Ω resistor.



The equivalent resistance is Req. = (10-1 + 10-1)-1 + 5 = 10Ω. So the current through the battery is I = 10V/10Ω = 1Ω. The current through the middle resistor is then I = ΔV/10 = (10 – 5∙1)/10 = 0.5A.

**Question 5.** Suppose we connect a battery (V = 10V) to two germanium resistor (ρGe = 0.46Ω·m), via two copper wires of negligible resistance. Suppose the left resistor has a length of ℓ1 = 1cm, and radius r1 = 0.5cm, while the right side one has a length ℓ2 = 2cm, and radius r2 = 1cm.



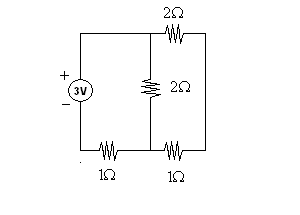
(a) What is the current in the circuit?

I = ΔV/Req. Req. = R1 + R2 = ρℓ1/A1 + ρℓ2/A2 = (0.46)(0.01)/(π×0.0052) + (0.46)(0.02)/(π×0.012) = 58.5Ω + 29.2Ω = 87.7 Ω. So I = 10/87.7 = 0.114 A

(b) What is the electric field in the right resistor?

The field in the right resistor is E = ΔV/Δs = IR2/ℓ2 = (0.029)(117)/0.02 = 170 N/C.

**Question 6**. What is the power dissipated across the upper 2Ω resistor?

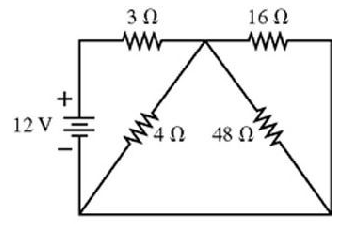


The 2Ω and 1Ω are in series to give 3Ω. Then this can be combined in parallel with the 2Ω resistor to give R = [1/3 + 1/2]-1 = 6/5 = 1.2Ω. And then this can be combined in series with the 1Ω to give Req. = 2.2Ω. Working backwads, across the 2.2Ω resistor is I = 3/2.2 = 1.36A.

So the current through the upper 2Ω resistor can be determined by using Kirchoff’s law for the outside loop. We have: 3 – i(2) – i(1) – (1.36)(1) = 0 → i = (3 – 1.36)/(2 + 1) = 0.55A.

And the power dissipated is: P = I2R = (0.54)2(2) = 0.58W.

**Question 6**.What is the power dissipated across the 16Ω resistor?



The equivalent resistance is Req. = (16-1 + 48-1 + 4-1)-1 + 3 = 6Ω. So the current through the battery is I = ΔV/Req. = 12/6 = 2A. Using Kirchoff’s voltage law around the outer loop we can say that 12 – 3(2) – 16∙I = 0 → I = 6/16 = 0.375A. And so the power dissipated across the 16Ω resistor is P = I2R = (0.375)2(16) = 2.25 W.